Supplementary Information
Quantifying the effect of temporal resolution in time-varying networks

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In this supplementary information appendix we cover some details left out of our main paper.

- Section 1 describes the evolution of a RW on an activity driven network.
- Section 2 provides a detailed derivation of Eq.(1) in the main text.
- Section 3 analyzes the random walk occupation probability when $\Delta t \to 0$.
- Section 4 analyzes the random walk occupation probability in the special case of time-varying dyadic networks ($m = 1$).
- Section 5 analyzes the random walk occupation probability in the special case of time-varying bipartite projected networks for $\Delta t \to 0$.
- Section 6 describes the datasets used in this work.
- And finally, Section 7 details our simulation results on real datasets.

1 RWs on activity driven networks

A RW on an activity driven network evolves as follows: Starting at node $V(t)$ a time $t\Delta t$, the walker takes a step at time $(t+1)\Delta t$ diffusing over a network $G_t(\Delta t)$, where $G_t(\Delta t)$ is the result of the union of all the edges generated in the interval $[t\Delta t, (t+1)\Delta t)$ and $t = 0, 1, \ldots$. We refer to Δt as the integrating time window of the network. In weighted aggregated networks, an integer edge weight shows the number of times the same edge appears during interval $[t\Delta t, (t+1)\Delta t)$ and, at each RW step, the walker chooses an edge with probability proportional to its weight. In unweighted aggregated networks, edges are either present, if they appear during interval $[t\Delta t, (t+1)\Delta t)$; otherwise the edge is absent. In the limit $\Delta t \to 0$ the RW process and the network evolve on the same timescale, with the walker moving as soon as an edge appears This limit has been studied analytically [1] in the framework of activity driven networks [2], where each node is characterized by an activity rate describing the average edge creation rate of a node in the system

RW stationarity and uniqueness conditions

An important requirement for the RW to be stationary and ergodic is for the network to be connected in time. A T-connected [3] time-varying network is a network that the aggregated network over $\Delta t \to \infty$ forms a connected graph (not necessarily fully connected). Consider some general stationary, ergodic, and T-conenected time-varying network with a fixed set of N nodes. From Theorem 3.1 of Figueiredo et al. [3] a RW on such network is stationary, and the stationary distribution is unique. To achieve this results we just need to translate the RW framework of Figueiredo et al. into our framework, which requires only setting parameter $\gamma \to \infty$ of the Figueiredo et al. RW, described in the paragraph after Definition 2.4).

2 Derivation of $Q_{a|a'}(\Delta t)$

Let $N \gg 1$ denote the total number of nodes in the graph. Let Ω be the set of all possible activity rates. There are no restrictions on the sample space Ω, which can be a discrete subset or a collection of continuous subsets. E.g., $\Omega = \{0.1, 0.2, 0.3\}$, another example is $\Omega = \{(0, 0.5), (0.8, 1)\}$, and our likely scenario $\Omega = (0, 1)$. Let $\mathrm{dF}(a)$ denote the probability that a randomly chosen node has activity a. We write $\mathrm{dF}(a)$ instead of the more familiar density function $p(a)da$ because da may not be well defined if Ω is discontinuous or discrete. Let $V(t)$ be the node that the RW is at time $t\Delta t$ and let $A(t)$ denote the activity of node $V(t)$. If $A(t) = a$ then the number of times $V(t)$ is active during interval Δt, denoted $K_{\Delta t,a}$, is Poisson distributed in an activity driven network, i.e.,

$$\mathrm{P}[K_{\Delta t,a} = k] = \frac{(a\Delta t)^k}{k!} \exp(-a\Delta t).$$

Let $H_{\Delta t,a}$ be the number of times any other node in the network connects to $V(t)$ then

$$\mathrm{P}[H_{\Delta t,a} = h] \approx \frac{(m\langle a\rangle\Delta t)^h}{h!} \exp(-m\langle a\rangle\Delta t),$$

where above we use the fact that $N \gg 1$ so that $m(N\langle a\rangle - a)/(N - 1) \approx m\langle a\rangle$. Thus, for all $a, a' \in \Omega$,

$$\begin{aligned}
&\mathrm{dP}[A((n+1)\Delta t) = a \,|\, A(n\Delta t) = a'] = \\
&= \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \mathrm{dP}[A((n+1)\Delta t) = a \,|\, A(n\Delta t) = a',\, K_{\Delta t,A(n\Delta t)} = k,\, H_{\Delta t,A(n\Delta t)} = h] \\
&\quad \times \mathrm{P}[K_{\Delta t,A(n\Delta t)} = k\,,\, H_{\Delta t,A(n\Delta t)} = h \,|\, A(n\Delta t) = a'] \\
&= \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \left(\frac{U_m(N;k)}{U_m(N;k) + h + \epsilon} \mathrm{dF}(a) + \frac{h}{U_m(N;k) + h + \epsilon} \frac{a\,\mathrm{dF}(a)}{\langle a\rangle} + \frac{\epsilon}{U_m(N;k) + h + \epsilon} \delta(a - a') \right) \\
&\quad \times \mathrm{P}[K_{\Delta t,a'} = k]\mathrm{P}[H_{\Delta t,a'} = h],
\end{aligned}$$

where $\epsilon \to 0$ and $U_m(N;k)$ is a the number of blue nodes in the graph after the following node coloring process:

1. Start with a set of N nodes all colored white;

2. pick m randomly sampled nodes chosen without replacement and color them blue;
3. repeat step 2 exactly k times;
4. $U_m(N;k)$ is the total number of blue nodes in the set.

This problem is known as the coupon collector problem with batch selections. Note that Pólya's urn model is a different model. In Pólya's model when a node of a particular color is drawn, that node is put back along with a *new* node of the same color, i.e., the size of the graph increases at each round.

In the regime where the network is large enough in respect to Δt, $N \gg 1$, such that with high probability an active node does not randomly choose the same neighbor twice in an interval Δt – that is, a time-varying edge appears only once in an interval Δt –, or more formally $P[U(N;k) < mK_{\Delta t,a}] \approx 0$, $\forall a \in \Omega$, yields

$$Q_{a|a'}(\Delta t) = \lim_{\epsilon\to 0} \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \left(\frac{m\,k}{m\,k+h+\epsilon} \mathrm{dF}(a) + \frac{h}{m\,k+h+\epsilon} \frac{a\,\mathrm{dF}(a)}{\langle a \rangle} + \frac{\epsilon}{m\,k+h+\epsilon} \delta(a-a') \right) \quad (1)$$
$$\times \frac{(a'\Delta t)^k}{k!} \exp(-a'\Delta t) \times \frac{(m\langle a\rangle \Delta t)^h}{h!} \exp(-m\langle a\rangle \Delta t)\,.$$

Eq. (1) is also valid for N small if the aggregated network has weights representing the number of times the same edge appears during the interval Δt. In such weighted aggregated network the random walk chooses a neighbor with probability proportional to the neighbor's edge weight. We take an in-depth look at RWs on weighted aggregated networks in the special case $m = 1$ shown in Section 4.

3 Special Case 1: $\Delta t \to 0$

Assumption 1. We assume $N \gg 1$ large enough such that $\mathrm{P}[U_m(N,k) < mK_{\Delta t,a}] \approx 0$, $\forall a \in \Omega$.

Recall that we defined $Q_{a|a'} \equiv \mathrm{dP}[A((n+1)\Delta t) = a \,|\, A(n\Delta t) = a']$. For all $a, a' \in \Omega$, $n \geq 0$,

$$Q_{a|a'}(\Delta t) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \left(\frac{km}{km+h+\epsilon} \mathrm{dF}(a) + \frac{h}{km+h+\epsilon} \frac{a\,\mathrm{dF}(a)}{\langle a\rangle} + \frac{\epsilon}{km+h+\epsilon} \delta(a-a') \right)$$
$$\times \mathrm{P}[K_{\Delta t,a'} = k] \mathrm{P}[H_{\Delta t,a'} = h]. \quad (2)$$

Definition 1: Define $o(x)$ as an undefined function of x such that $\left|\frac{o(x)}{x}\right| \to 0$ as $x \to 0$.

The probabilities the a node with activity a is active under Assumption 1 are:

- $\mathrm{P}[K_{\Delta t,a'} \geq 1, H_{\Delta t,a} \geq 1] = o(\Delta t)$,
- $\mathrm{P}[K_{\Delta t,a'} = 1, H_{\Delta t,a} = 0] = a'\Delta t + o(\Delta t)$,
- $\mathrm{P}[K_{\Delta t,a'} = 0, H_{\Delta t,a} = 1] = m\langle a\rangle\Delta t + o(\Delta t)$,
- $\mathrm{P}[K_{\Delta t,a'} = 0, H_{\Delta t,a} = 0] = 1 - (a' + m\langle a\rangle)\Delta t + o(\Delta t)$,

Substituting the above equalities into (2) yields

$$Q_{a|a'}(\Delta t) = (1 - (a' + m\langle a\rangle)\Delta t)\delta(a-a') + \mathrm{dF}(a)a'\Delta t + \frac{a\,\mathrm{dF}(a)}{\langle a\rangle} m\langle a\rangle\Delta t \quad (3)$$

RW stationary distribution

Define $\rho_a(n) \equiv \mathrm{dP}[A(n\Delta t) = a]/(N\,\mathrm{dF}(a))$ as the RW occupation probability. Define $\mathrm{d}\rho_a(n+1) \equiv \rho_a(n+1) - \rho_a(n)$ as the increase in probability from time $n\Delta t$ to time $(n+1)\Delta t$ that the walker is in a node with activity a. The quantity $\mathrm{d}\rho_a(n+1)$ is the probability that a walker that was at a node with activity a' *and* moved to a node with activity a minus the probability that the walker was in a node with activity a *and* moved to a node with activity a', integrated over all $a' \in \Omega\backslash\{a\}$. More formally,

$$\begin{aligned}
\mathrm{d}\rho_a(n+1) =& \frac{1}{N\mathrm{dF}(a)} \int_{a'\in\Omega\backslash\{a\}} \mathrm{dP}[A((n+1)\Delta t) = a,\, A(n\Delta t) = a'] \qquad (4)\\
&- \mathrm{dP}[A((n+1)\Delta t) = a',\, A(n\Delta t) = a]\\
=& \frac{1}{N\mathrm{dF}(a)} \int_{a'\in\Omega} \mathrm{dP}[A((n+1)\Delta t) = a \,|\, A(n\Delta t) = a']\mathrm{dP}[A(n\Delta t) = a']\\
&- \mathrm{dP}[A((n+1)\Delta t) = a' \,|\, A(n\Delta t) = a]\mathrm{dP}[A(n\Delta t) = a] \qquad (5)\\
=& \frac{1}{N\mathrm{dF}(a)} \int_{a'\in\Omega} \left(\mathrm{dF}(a)a'\Delta t + \frac{a\,\mathrm{dF}(a)}{\langle a\rangle} m\,\langle a\rangle\,\Delta t\right) \mathrm{dP}[A(n\Delta t) = a']\\
&- \int_{a'\in\Omega} \left(\mathrm{dF}(a')a\Delta t + \frac{a'\,\mathrm{dF}(a')}{\langle a\rangle} m\,\langle a\rangle\,\Delta t\right) \mathrm{dP}[A(n\Delta t) = a]\\
=& \frac{1}{N\mathrm{dF}(a)} \Delta t \left(\int_{a'\in\Omega} \mathrm{dF}(a)a'\,\mathrm{dP}[A(n\Delta t) = a'] + \int_{a'\in\Omega} a\,\mathrm{dF}(a)m\mathrm{dP}[A(n\Delta t) = a'] \right.\\
&\left. - \int_{a'\in\Omega} \mathrm{dF}(a')a\,\mathrm{dP}[A(n\Delta t) = a] - \int_{a'\in\Omega} a'\,\mathrm{dF}(a')m\,\mathrm{dP}[A(n\Delta t) = a] \right),
\end{aligned}$$

where in (5) we use the fact that $\mathrm{dP}[A((n+1)\Delta t) = a,\, A(n\Delta t) = a] - \mathrm{dP}[A((n+1)\Delta t) = a,\, A(n\Delta t) = a] = 0$ to add $\{a\}$ to the integral. Thus,

$$\begin{aligned}
\frac{\mathrm{d}\rho_a(n+1)}{\Delta t} =& \mathrm{dF}(a) \int_{a'\in\Omega} a'\,\mathrm{dP}[A(n\Delta t) = a'] + a\,\mathrm{dF}(a)m \int_{a'\in\Omega} \mathrm{dP}[A(n\Delta t) = a']\\
&- a\,\mathrm{dP}[A(n\Delta t) = a] \int_{a'\in\Omega} \mathrm{dF}(a') - \mathrm{dP}[A(n\Delta t) = a] \int_{a'\in\Omega} a'\,\mathrm{dF}(a')m. \qquad (6)
\end{aligned}$$

Using our definition of $\rho_a(n) \equiv \mathrm{dP}[A(n\Delta t) = a]/(N\mathrm{dF}(a))$, Eq. (6) yields

$$\begin{aligned}
\frac{N\mathrm{dF}(a)\mathrm{d}\rho_a(n+1)}{\Delta t} =& \mathrm{dF}(a) \int_{a'\in\Omega} a'\,N\mathrm{dF}(a')\rho_{a'}(n) + a\,\mathrm{dF}(a)m \int_{a'\in\Omega} N\mathrm{dF}(a')\rho_{a'}(n)\\
&- a\,\rho_a(n)N\mathrm{dF}(a) \int_{a'\in\Omega} \mathrm{dF}(a') - \rho_a(n)N\mathrm{dF}(a) \int_{a'\in\Omega} a'\,\mathrm{dF}(a')m.
\end{aligned}$$

Dividing both sides by $N\mathrm{dF}(a)$ yields

$$\begin{aligned}\frac{\mathrm{d}\rho_a(n+1)}{\Delta t} &= \int_{a'\in\Omega} a'\,\mathrm{dF}(a')\rho_{a'}(n) + a\,m\int_{a'\in\Omega}\mathrm{dF}(a')\rho_{a'}(n)\\ &\quad - a\,\rho_a(n)\int_{a'\in\Omega}\mathrm{dF}(a') - \rho_a(n)\int_{a'\in\Omega} a'\,\mathrm{dF}(a')m\\ &= \int_{a'\in\Omega} a'\,\mathrm{dF}(a')\rho_{a'}(n) + a\,m\int_{a'\in\Omega}\mathrm{dF}(a')\rho_{a'}(n) - \rho_a(n)(a+\langle a\rangle m). \end{aligned} \tag{7}$$

Because the RW is stationary and ergodic, as walker progresses, i.e., $n \gg 1$, $\rho_a(n)$ reaches a stationary distribution. More precisely,

$$\lim_{n\to\infty}\mathrm{d}\rho_a(n) = 0.$$

Substituting the above limit in (7) and defining the stationary occupation probability $\rho_a \equiv \lim_{n\to\infty}\rho_a(n)$ we get the following flow balance equations

$$\int_{a'\in\Omega} a'\,\mathrm{dF}(a')\rho_{a'} + a\,m\int_{a'\in\Omega}\mathrm{dF}(a')\rho_{a'} = \rho_a(a-\langle a\rangle m).$$

Define $\langle\rho_a\rangle = \int_{a\in\Omega} a\,\rho_a\mathrm{dF}(a)$ and then we simplify the above to

$$\rho_a = \frac{\langle\rho_a\rangle + am}{a+\langle a\rangle m}. \tag{8}$$

To obtain $\langle\rho_a\rangle$ observe that

$$\begin{aligned}\langle\rho_a\rangle &= \int_{a\in\Omega} a\rho_a\mathrm{dF}(a) = \int_{a\in\Omega}\frac{a\left(\langle\rho_a\rangle + am\right)}{a+\langle a\rangle m}\mathrm{dF}(a)\\ &= \langle\rho_a\rangle\int_{a\in\Omega}\frac{a}{a+\langle a\rangle m}\mathrm{dF}(a) + \int_{a\in\Omega}\frac{a^2m}{a+\langle a\rangle m}\mathrm{dF}(a)\\ \langle\rho_a\rangle &= \frac{m\beta_2}{1-\beta_1},\end{aligned}$$

where

$$\beta_i = \int_{a\in\Omega}\frac{a^i}{a+\langle a\rangle m}\mathrm{dF}(a).$$

We note in passing that Eq. (8) is exactly the result in Perra et al. [2].

4 Special Case 2: $m = 1$ (dyadic time-varying network)

Consider a Poisson process where edges arrive to node $V(t)$ with rate $a' + \langle a\rangle$ and let $R_{\Delta t}$ be the total number of edges attached to node $V(t)$ during time window $(t\Delta t, (t+1)\Delta t]$. Note that the network is assumed stationary and thus $R_{\Delta t}$ does not depend on t. Moreover, $R_{\Delta t}$ is Poisson distributed with rate $(a' + \langle a\rangle)$,

$$P[R_{\Delta t} = r] = \frac{((a'+\langle a\rangle)\Delta t)^r}{r!}e^{-(a'+\langle a\rangle)\Delta t}.$$

Note that $R_{\Delta t}$ does not depend on t as the network process is stationary.

Next we randomly assign edges one of two of the following **types**: an edge is of type **passive** with probability $a'/(a' + \langle a \rangle)$ and of type **active** with probability $\langle a \rangle/(a' + \langle a \rangle)$. From the infinite divisibility property of the Poisson distribution, the the number of *passive* and *active* edges are Poisson distributed with parameters $a'\Delta t$ and $\langle a \rangle \Delta t$, respectively.

Edge types & weighted aggregated networks. The above model does not describe a network but rather just **edge arrivals** and **type assignments** at a node. Fortunately, such description suffices in activity driven networks. This happens because at the next RW step, the network reconstructs itself, allowing us to treat the coupled RW and network dynamics as a simple renewal process. Interestingly, the above model already considers multiple appearances of the same edge as long as the aggregated network is represented as a weighted aggregated network.

Static network representations of time-varying networks can weighted or unweighted. In **weighted aggregated networks**, edges in $G_t(\Delta t)$, where $G_t(\Delta t)$ is the result of the union of all the edges generated in the interval $[t\Delta t, (t+1)\Delta t)$ (see Figure 1 of our main paper), have integer weights that represent the number of times the edge appears during interval $[t\Delta t, (t+1)\Delta t)$. In **unweighted aggregated networks**, edges are unweighted. An edge is present in $G_t(\Delta t)$ if it appears one or more times during interval $[t\Delta t, (t+1)\Delta t)$; otherwise the edge is not present. Throughout this work we consider unweighted aggregated networks. However, one of our main results, namely Section 4 result on the random walk occupation probability on activity driven networks with $m = 1$ concurrent edge creations, can be readily applied to weighted network representations as well.

From the point of view of the walker, a weighted network with integer edge weights has an equivalent multigraph. A multigraph is a graph that allows multiple edges between nodes. The multigraph is constructed as follows: for each edge (u, v) with weight $w \in \mathbb{N}$ in the weighted graph add w edges (u, v) in the multigraph. A RW on a multigraph, just like a RW on a weighted graph, selects a destination endpoint with probability proportional to the number of edges to that destination (its weight in the weighted graph). In the regime where the probability that a node connects to the same edge twice is close to zero – e.g., $N \gg 1$ is large enough in respect to Δt – then the weighted graph is a simple 0-1 graph with high probability (and thus equivalent to an unweighted network). In what follows we assume that the network is a multigraph graph, which encompasses the in special scenario of 0-1 graphs.

Derivations. Recall that the walker randomly chooses one destination out of the $R_{\Delta t}$ edges in the multigraph. Because the type of the first edge – passive or active – is selected randomly, the random walk choice of edge is statistically equivalent to committing to always choose the first edge before knowing its type. We wish to remind the reader that $V(t)$ has activity rate a'. The probability that the first edge has a *passive* destination is $a'/(a' + \langle a \rangle)$ and the probability that it has an *active* destination is $\langle a \rangle/(a' + \langle a \rangle)$. The probability that $V(t)$ has no edge after a time window of size Δt is $\zeta_{a',\Delta t} = e^{-(a'+\langle a \rangle)\Delta t}$. Then, the probability that the walker moves from $V(t)$ to an active destination with activity a is $(1 - \zeta_{a',\Delta t})\langle a \rangle/(a' + \langle a \rangle) \times a\mathrm{dF}(a)/\langle a \rangle = (1 - \zeta_{a',\Delta t})a\mathrm{dF}(a)/(a' + \langle a \rangle)$. The probability that the walker moves from $V(t)$ to a passive destination with activity a is $(1 - \zeta_{a',\Delta t})a'/(a' + \langle a \rangle) \times \mathrm{dF}(a) = (1 - \zeta_{a',\Delta t})a'\mathrm{dF}(a)/(a' + \langle a \rangle)$. The probability that the walker stays in $V(t)$ is $\zeta_{a',\Delta t}$, which is the probability that there are no edges out of $V(t)$. Thus, summing all these factors we obtain the probability that the walker moves from a node with activity a' to a node with activity a:

$$Q_{a|a'}(\Delta t) = \left(\frac{a\mathrm{dF}(a)}{a' + \langle a\rangle} + \frac{a'\mathrm{dF}(a)}{a' + \langle a\rangle}\right)(1 - \zeta_{a',\Delta t}) + \delta(a' - a)\zeta_{a',\Delta t} \tag{9}$$

$$= \frac{a + a'}{a' + \langle a\rangle}\mathrm{dF}(a)(1 - \zeta_{a',\Delta t}) + \delta(a' - a)\zeta_{a',\Delta t}. \tag{10}$$

The occupation probabilities $\{\rho_a\}_{\forall a\in\Omega}$, are the unique solution to the fixed point set of Chapman-Kolmogorov equations

$$\rho_a = \frac{1}{\mathrm{dF}(a)} \int_{a'\in\Omega} Q_{a|a'}(\Delta t)\rho_{a'}\mathrm{dF}(a')\,, \quad \forall a \in \Omega\,. \tag{11}$$

Uniform occupation probability when $\Delta t \to 0$. Recall that $\zeta_{a,\Delta t} = e^{(a+\langle a\rangle)\Delta t}$. For small aggregating windows

$$\lim_{\Delta t\to 0} \zeta_{a,\Delta t} \to 1 \quad , \quad \forall a \in \Omega.$$

Thus, the first term of Eq. (9) is zero. Then $Q_{a|a'}(\Delta t) = \mathrm{dF}(a)$ and $\rho_a = 1/N$.

occupation probability when $\Delta t \gg 1$. For large aggregation windows, $\Delta t \gg 1$, the value of $\zeta_{a,\Delta t} \approx 0$, $\forall a \in \Omega$, and thus the second term of Eq. (9) is zero. In this scenario $Q_{a|a'}(\Delta t) = C\,(a + \langle a\rangle)\mathrm{dF}(a)$, where $C = 1/2\langle a\rangle$. The fixed point solution of Eq. (11) is $\rho_a = (a + \langle a\rangle)/2N\langle a\rangle$.

5 RW behavior on time-varying bipartite projection networks when $\Delta t \to 0$

As $\Delta t \to 0$ network nodes are either isolated or belong to a clique. For instance, in the co-citation network assume we measure the time that authors submit their work to the journal at time t. Authors cannot submit work simultaneously to the same journal – although authors may to submit multiple articles at short bursts so that they end up in the same journal volume. At time t an author is either isolated – when the author did not submit a paper at time t – or connected in a clique formed by the co-authors of the paper submitted at time t. We can then use Theorem 3.4 of Figueiredo et al. [3] which shows that a RW on any (stationary, ergodic, and T-connected) time-varying network whose snapshots are cliques has uniform occupation probability, that is, $\rho_a = 1/N$.

6 Dataset Details

In this study we considered two different empirical time-varying networks. The collaborations in the journal "Physical Review Letter" (PRL) published by the American Physical Society [4], and the Yahoo! music dataset. Here we present a detailed description of both datasets.

6.1 PRL dataset

In this dataset the network representation considers each author of an article in PRL as a node. Undirected links connect authors that collaborate in the same article. We focus just on small collaboration filtering out all the articles with more than 10 authors. We consider the period between 1958 and 2006. The datasets contains 80,554 authors and 66,892 articles. The smallest timescale available is $\Delta t = 1$ day.

6.2 Yahoo! Music dataset

This database contains 4.6×10^5 songs rated by 199,719 users of Yahoo! users collected in the course of 6 months [5]. User activity is recorded at a time resolution of seconds. Two songs form an edge if they are rated by the same user in the same time window Δt.

7 Simulation on Real Datasets

We obtain the empirical walker occupation probability, ρ_a, as follows. Construct the transition probability matrix P_t associated to the RW on the t-th aggregated network $G_t(\Delta t)$, $t = 0, 1, \ldots, \lfloor T/\Delta t \rfloor$, where T is the time of the last event in the dataset. The empirical RW occupation probability is obtained by multiplying the matrices $P_0 \, P_1 \cdots P_n$ and left multiplying the result by the vector $(1/N, \ldots, 1/N)$, which gives equal probability that for walker to start at any node. We note in passing that similar results are obtained when the walker starts at a handful of high activity nodes.

Note that in projected bipartite networks, an increase in Δt by a factor of α, $\Delta t' = (1+\alpha)\Delta t$ does not correspond to an increase in activity by $(1+\alpha)$. This is because while there $(1+\alpha)$ links from the agents to the objects, the number of connections between agents in the projected network does not necessarily increase by $(1+\alpha)$. In order to take this non-trivial projection effect into account, we rescale our Δt as to best fit the observed data.

Obtaining $dF(a)$ from the data: Let $\mathrm{F}(a)$ be the fraction of nodes with activity greater or equal than a. By definition $\lim_{\epsilon \to 0} \mathrm{dF}(a) = \mathrm{F}(a) - \mathrm{F}(a+\epsilon)$. But choosing ϵ too small creates an unecessary computational burden for our equations. In order to speed up computation to a matter of seconds in a modern machine, we choose ϵ as a function of a, ϵ_a, such that ϵ_a is the smallest value such that $\mathrm{F}(a) - \mathrm{F}(a+\epsilon_a) > 10^{-2}$. However, it is worth emphasizing that our equations are amenable to any choice of $\epsilon > 0$.

The results of Figures 3 and 4 in our main paper were obtained through the above procedure applied to the simulations of $\Delta t = 1$ for both datasets. In what follows we show the empirical $\mathrm{F}(a)$ as a function of the activity a of our datasets. Figure S1 plots the empirical $\mathrm{F}(a)$ against a of the PRL author activity for different aggregation windows, $\Delta t \in \{$one day, ten days, two months, 6 months$\}$. And Figure S2 plots the empirical $\mathrm{F}(a)$ against a of the Yahoo! Music song activity for different aggregation windows, $\Delta t \in \{$one second, one minute, one hour, six hours, and one day$\}$. In the Yahoo! Music dataset we observe that the empirical values of $\mathrm{F}(a)$ for $\Delta t \in \{$one minute, one hour, six hours, and one day$\}$ are similar. However, the empirical $\mathrm{F}(a)$ for $\Delta t = 1$ is significantly different from the empirical $\mathrm{F}(a)$ for other values of Δt.

To evaluate the impact of the empirical $\mathrm{dF}(a)$ for different choices of Δt in the results of Figure 4 of our main paper, we recompute Eq. (11) using the empirical $\mathrm{dF}(a)$ obtained from $\Delta t = 60$ instead of the empirical $\mathrm{dF}(a)$ obtained from $\Delta t = 1$ as in the original figure. Figure S3 shows our results. We note that the main difference between the results obtained in Figure S3 and the ones in Figure 4 of our main paper are concentrated on low activity nodes, which are better modeled by the empirical $\mathrm{dF}(a)$ from $\Delta t = 1$. Comparing again the figures for high activity nodes shows that our analytical results are robust to the choice of Δt when extracting the empirical $\mathrm{dF}(a)$. Our final observation is

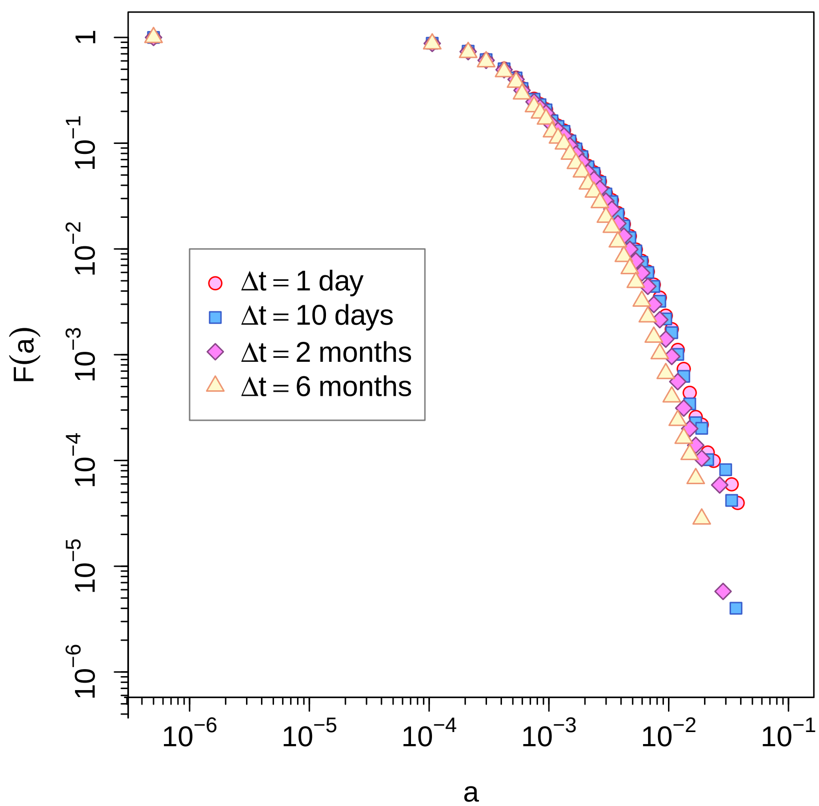

Figure S1: **(PRL dataset)** $F(a)$ for Δt of one day, ten days, two months, six months.

then that in our datasets choosing the lowest resultion of Δt to obtain the empirical dF(a) works best.

References

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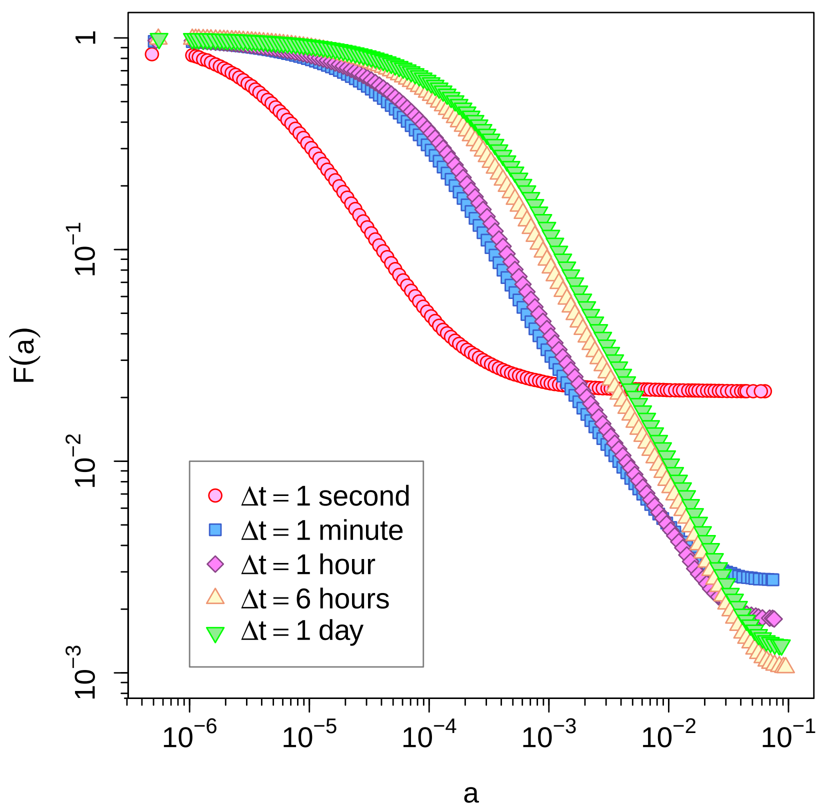

Figure S2: **(Yahoo! dataset)** $F(a)$ for Δt of one second, one minute, one hour, six hours, and one day.

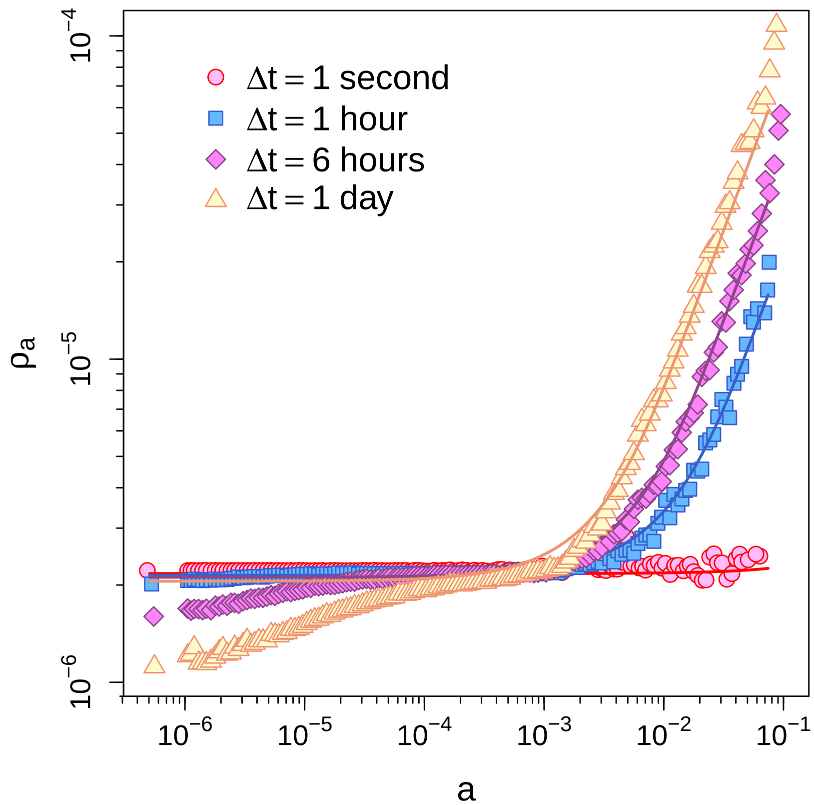

Figure S3: Occupation probability ρ_a of a RW at the end of the simulation as a function of node activity. This figure differs from Figure 4 of our main paper in that dF(a) in the main paper is obtained from Δt of one second and in the above figure dF(a) is obtained from Δt of one hour. The points are the values of ρ_a on the time-varying graph of Yahoo! song ratings for different integrating windows Δt of one second, one hour, six hours, and one day. The solid lines are the numerical solution of Eq. (11). The errors bars are not visible in this case.

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